Spinorial space-time and Friedmann-like equations (I)

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Abstract

The use of a spinorial space-time naturally leads to unconventional properties for the space curvature term in Friedmann-like equations. It therefore suggests a major modification of the Standard Cosmology based on general relativity. In the new Cosmology thus introduced, the contribution of the space curvature to the value of H^2 (H = Lundmark Lematre Hubble, LLH, constant) is positive definite independently of the apparent sign of this curvature, and has a much larger value than the standard curvature term. Then, a cosmological constant is no longer needed. The spinorial space-time also generates a privileged space direction compatible with WMAP and Planck data. Taking into account the naturally dominant role of the new curvature term, it is tempting to conjecture that the product H t (t = cosmic time = age of the Universe) tends asymptotically to 1 in the long-term evolution of our Universe up to possible small corrections. We remind here our previous work on the subject and further discuss some cosmological implications of the spinorial space-time including space curvature.

1 Introduction

In [1, 2], we suggested to replace the standard real four-dimensional space-time by the complex two-dimensional one actually felt by spin-1/2 particles. Then, the standard rotation group SO(3) for space dimensions is replaced by the spinorial SU(2) allowing spin-1/2 particles to be representations of the actual rotation group.

Cosmologically comoving frames naturally appear in the spinorial space-time, and conventional relativity may just correspond to the space-time structure felt locally by standard matter at low energy [3]. At the stage considered here, the spinorial space-time does not require the use of any critical speed.

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The spinorial space-time automatically generates a privileged space direction for each co-moving observer [4, 5] and can therefore, together with parity violation, potentially explain the asymmetry reported by Planck [6] confirming previous WMAP observations.

From a cosmic space-time position SU(2) spinor ξ , the SU(2) scalar $\mid \xi \mid^2 = \xi^\dagger \xi$ where the dagger stands for hermitic conjugate, defines a positive cosmic time $t = \mid \xi \mid$ leading in particular to a naturally expanding Universe where inflation is no longer needed [1, 7]. Other definitions of cosmic time based on the same scalar produce in practice similar results [7]. The point $\xi = 0$ plays the role of a cosmic time origin (birth of the Universe) and the $\mid \xi \mid = t_0$ hypersphere describes the space at the time t_0. Because of the pre-existing spinorial geometry, this S^3 hypersphere has an additional structure that will allow to obtain the privileged space direction associated to each comoving observer.

SU(2) transformations of a cosmic spinor ξ correspond to space translations where the space-time origin $\xi = 0$ remains unchanged. Relative space coordinates are defined in this way, whereas space rotations of these coordinates around a fixed space-time point ξ_0 are SU(2) transformations acting on the SU(2) group itself [2, 5]. The distance between two points on the space hypersphere is given by t_0 times the angle θ separating the two points in a circle of radius t_0 . As a 2π SU(2) rotation changes the sign of the spinor ξ_0 , the value $\theta = 2\pi$ corresponds to the antipodal space-time position $-\xi_0$.

The privileged space direction associated to a cosmic spinor ξ is then defined by the set of SU(2) transformations (translations) that leave ξ invariant up to a global complex phase : $\xi' = U\xi = exp\,(i\phi)\,\xi$ where U is a SU(2) transformation, ϕ is real and $exp\,(i\phi)$ is a complex phase factor. Such a definition is invariant under SU(2) transformations and comoving time evolution. An equivalent definition (tangent direction) is provided by the vector $\mid \xi \mid^{-2} \xi^\dagger \vec{\sigma} \xi$ where $\vec{\sigma}$ is the vector formed by the three Pauli matrices. The value of $\mid \xi \mid^{-2} \xi^\dagger \vec{\sigma} \xi$ is common to all points lying on the privileged space trajectory.

An even stronger prediction of the spinorial space-time [7, 8] is that the contribution of the space curvature to the value of H^2 in Friedmann-like equations radically changes and becomes dominant. Its value in the absence of matter and dark energy is t^{-2} , positive and independent of the sign of the observed space curvature. This important property, that suggests a new approach to Cosmology, will be further discussed in **Section 3**.

2   Our February 1997 paper arXiv:physics/9702026

The cosmological implications of the spinorial space-time, including the universe expansion and the privileged space direction, appeared already in our early papers on the subject. In reference [2], we wrote:

http://arxiv.org/abs/physics/9702026
http://arxiv.org/pdf/physics/9702026v1.pdf

(...)

2. THE SPINORIAL SPACE-TIME

Instead of four real numbers, we take space-time to be described by two complex numbers, the components of a $SU(2)$ spinor. From a spinor ξ , it is possible to extract a $SU(2)$ scalar, $\mid \xi \mid^2 = \xi^\dagger \xi$ (where the dagger stands for hermitic conjugate), and a vector $\vec{\mathbf{z}} = \xi^\dagger \vec{\sigma} \xi$, where $\vec{\sigma}$ is the vector formed by the Pauli matrices. In our previous papers on the subject [4 , 5] , we proposed to interpret $t = \mid \xi \mid$ as the time. If the spinor coordinates are complex numbers, one has: $z = t^2$ where z is the modulus of $\vec{\mathbf{z}}$. It does not seem possible to interpret $\vec{\mathbf{z}}$ as providing the space coordinates: one coordinate, corresponding to an overall phase of the spinor, is missed by t and $\vec{\mathbf{z}}$. Therefore, a different description of space seems necessary in this approach.

Interpreting t as the time has at first sight the drawback of positive-definiteness and breaking of time reversal, but this can be turned into an advantage if t is interpreted as an absolute, cosmic time (geometrically expanding Universe). An arrow of time is then naturally set, and space-time geometry incorporates the physical phenomenon of an expanding Universe. As space translations and rotations are by definition transformations leaving time invariant, the space coordinates should be built by considering the polar coordinates in the hypersphere (i.e. the spherical hypersurface) of constant time, $\mid \xi \mid = t_0$ where t_0 is a value of time [4 , 5]. On this hypersphere, a point ξ can be decribed as:

$$\xi = U\xi_0 \tag{1}$$

where U is a $SU(2)$ transformation and ξ_0 a constant spinor (hereafter identified with the observer position) on the sphere $t = t_0$. Writing:

$$U = exp\ (i/2\ \ t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{x}})\ \equiv U(\vec{\mathbf{x}}) \tag{2}$$

the vector $\vec{\mathbf{x}}$, with $0 \leq x$ (modulus of $\vec{\mathbf{x}}$) $\leq 2\pi t_0$, can be interpreted as the position vector at constant time t_0 . (...)

It is obvious that, under a $SU(2)$ transformation V , U transforms into VUV^{-1} (the vector representation) if $\vec{\sigma}.\vec{\mathbf{x}}$ transforms into $V\ \vec{\sigma}.\vec{\mathbf{x}}\ V^{-1}$ (the vector linear representation). The vector $\vec{\mathbf{v}}$ obtained from the equation $V = exp\ (i/2\ \ \vec{\sigma}.\vec{\mathbf{v}})$ defines the rotation axis and angle in correspondence with $SO(3)$ rotations. $\vec{\mathbf{x}}$ provides the space coordinates and transforms like a $SO(3)$ real vector (...) . The observer can thus feel a vector space in a spinorial space-time.

The cosmic time scale given by the radius of the Universe does not correspond to the local time scale of physical processes at time t , which depends on the local vacuum dynamics and varies with cosmic time. (...)

(end of quote from [2])

The references [4,5] cited in [2] correspond to [1] of the present paper. The vector $\vec{\mathbf{z}} = \xi^\dagger \vec{\sigma} \xi$ provides the privileged space direction. As discussed with more detail in [4, 5] and in [8], the spinorial space-time thus introduced automatically generates the LLH law and the $H\ t = 1$ relation where t (cosmic time) is the age of the Universe and H the usual constant describing the ratio between relative velocities and distances among comoving observers. This law arises in a purely geometric way before introducing matter, relativity and gravitation.

3 Spinorial space-time and space curvature

This subject has been recently dealt with in [7, 8]. We further discuss here some specific points concerning the role of the spinorial space-time in Friedmann-like equations [9].

Let us consider the standard Friedmann equation:

$$H^2 \; = \; 8\pi G\rho/3 \; - \; kR^{-2}\,c^2 \; + \; \Lambda\,c^2/3 \tag{3}$$

where $H \; = \; a_s^{-1}\,da_s/dt$ is the LLH constant, a_s the scale factor, G the gravitational constant, ρ the energy density, c the speed of light, $k\;R^{-2}$ the curvature parameter, R the present curvature distance scale of the Universe (the curvature radius, and possibly the radius of the Universe, for $k=1$) and Λ the cosmological constant.

The relevant spinorial space-time equation for $\rho = 0$ and $\Lambda = 0$ just replaces the curvature term $-\;kR^{-2}\,c^2$ by t^{-2} where t is the cosmic time (age of the Universe). The $H\,t=1$ law is an immediate consequence of the spinorial geometry where straight lines through the origin correspond to cosmologically comoving frames. This is a substantial change with respect to usual patterns, as the standard curvature term is replaced by a much larger one and with a positive-definite sign irrespectively of the sign of k .

Furthermore, at the present stage only a cosmic time scale exists and plays also the role of an effective distance scale with the spherical space structure naturally suggested by the spinorial space-time geometry. No distinct distance scale or critical speed has yet been introduced. It is therefore possible to redefine the spatial distance at cosmic scale as seen by a comoving observer at a point ξ_0 on the $\xi \; = \; t$ hypersphere. Then, an effective hyperbolic curvature can replace the initial spherical one.

Contrary to the hyperspherical configuration, that is common to all comoving observers, such a hyperbolic structure, if ever relevant, would have a privileged comoving observer. The spinorial structure of space-time is more fundamental and remains unchanged by this new definition of distances on the S^3 hypersphere.

3.1 Sending the antipodal point to an infinite distance from the observer

More precisely, if D is the cosmic distance on the space hypersphere corresponding to the relevant angular distance θ times t (see [4, 5] and [8]), a possible alternative to this natural definition would be as follows. We send to infinity, from the point of view of the spatial distance on the S^3 hypersphere, the antipodal point $-\;\xi_0$ ($\theta \; = \; 2\pi$) as seen by the comoving observer based at ξ_0 . Without modifying the underlying spinorial space-time, this transformation turns $D \; = \; \theta\,t$ into a new definition of distance, D' , such that:

$$D^{\;2} \; = \; 4\,\pi^2\;|\,\xi_0\,|^2\;D'^{\;2}\,(4\,\pi^2\;|\,\xi_0\,|^2\;+\;D'^{\;2})^{-1} \tag{4}$$

leading to:

$$D'^{\;2} \; = \; 4\,\pi^2\;|\,\xi_0\,|^2\;D^{\;2}\,(4\,\pi^2\;|\,\xi_0\,|^2\;-\;D^{\;2})^{-1} \tag{5}$$

or, in terms of the angular variable θ :

$$D'^{\,2} \;=\; 4\,\pi^2 \;\mid \xi_0 \mid^2 \;\theta^{\,2}\,(4\,\pi^2 \;-\; \theta^{\,2})^{-1} \tag{6}$$

Capital letters are used to design distances, as the differential of a distance will be considered below.

Then, the value $D \;=\; 2\,\pi \mid \xi_0 \mid\,(\theta \;=\; 2\,\pi)$ corresponds to $D' = \infty$ and the spherical curvature can become a hyperbolic one. As D' is also proportional to t , the LLH law and the $H\,t = 1$ equation remain unchanged by this redefinition of distance.

Redefining spatial distance in this way amounts to modify the standard orthogonal coordinates $x = t\,\sin\,(\theta/2)$ and $y = t\,\cos\,(\theta/2)$, where θ is the angular distance between the point considered and ξ_0 . Replacing x and y by suitable new coordinates x' and y' such that:

$$(dD')^2 \;=\; (dx)^2 \;+\; (dz)^2 \tag{7}$$

a hyperbolic geometry can be built without introducing any new time or distance scale. This would be the case writing:

$$x \;=\; \alpha \mid \xi_0 \mid\,[exp(w) \;-\; exp(-w)] \tag{8}$$

where α is a constant and w a new variable, and taking :

$$y \;=\; \alpha \mid \xi_0 \mid\,[exp(w) \;+\; exp(-w)] \tag{9}$$

so that

$$y^2 \;-\; x^2 \;=\; 4\,\alpha^2 \mid \xi_0 \mid^2 \tag{10}$$

and

$$(dD'/dw)^2 \;=\; 2\,\alpha^2 \mid \xi_0 \mid^2\,[exp(2w) \;+\; exp(-2w)] \tag{11}$$

allowing also to relate w and θ . With $w = 0$ for $\theta = 0$ and taking small values of both variables, one gets:

$$\mid \xi_0 \mid^2\,(d\theta/dw)^2 \;\simeq\; 4\,\alpha^2 \mid \xi_0 \mid^2 \tag{12}$$

So that $d\theta/dw \simeq 1$ for $\alpha = 1/2$.

This is just an example of a possible hyperbolic structure generated from the spinorial space-time and using $\mid \xi_0 \mid$ as the natural distance scale.

3.2   Other similar transformations

In order to generate a hyperbolic curvature, it is also possible to send to infinity, from the point of view of distance as seen by the observer, a whole circle of the spatial hypersphere writing, for instance:

$$D'^{\,2} \;=\; 4\,\pi^2 \mid \xi_0 \mid^2\;D^{\,2}\,(\,4\,\lambda\,\pi^2 \mid \xi_0 \mid^2 \;-\; D^{\,2})^{-1} \tag{13}$$

where $0 \;<\; \lambda \;<\; 1$.

Then, one gets:

- $D' = \infty$ for $D = 2\pi \, \lambda^{1/2} \mid \xi_0 \mid$

- an imaginary D' for larger values of D.

Such a structure would imply, in particular, that no signal traveling in space from comoving points at $D > 2\pi \, \lambda^{1/2} \mid \xi_0 \mid$ can ever reach the comoving observer at ξ_0 . However, this hidden region of the Universe remains connected to the observer through the spinorial coordinates.

3.3   Cosmological implications

As the new definitions of the spatial distance just considered remain equal to $\mid \xi_0 \mid$ times a function of θ, the relation $H = t^{-1}$ is preserved together with the spinorial space-time structure.

Thus, contrary to conventional schemes, the spinorial space-time can account for both spherical and hyperbolic space curvatures with the same (dominant) positive-definite curvature term in the Friedmann-like equation and the $H = t^{-1}$ law in the absence of matter and of a cosmological constant. Observation gives $H \, t$ around 0.96.

This specific property of the Cosmology based on the spinorial space-time clearly clearly indicates the presence of a more fundamental geometry than that usually considered in theories obtained from standard relativity. Friedmann cosmology and derived phenomenology should therefore be reformulated.

Similarly, nothing in the transition from spherical to hyperbolic curvature appears to remove the existence of a privileged space direction for each comoving observer.

A significant difference between the spherical and hyperbolic views of space in the spinorial space-time is that the spherical description is generated as a direct consequence of the mathematical spinorial structure and is the same for all comoving observers. This is not the case of the hyperbolic structure, that is specific to each observer even in the choice of the (antipodal) point sent to infinity.

With a hyperbolic view of distances, the distance between two points in the Universe depends on the observer, contrary to the hyperspherical picture using comoving frames.

4   Complementary remarks

As already emphasized in [7], the issues discussed in the *Post Scriptum* to [4] deserve a more detailed analysis and will be dealt with in forthcoming articles. We have pointed out here and in [7, 9] an additional new property of the spinorial space-time : the fact that it can actually describe both spherical and hyperbolic space geometries with the same cosmological weight in Friedmann-like equations and the same law relating H and t.

As the spinorial space-time is likely to be the expression of a new fundamental vacuum structure at a more fundamental level than standard quantum field theory, most conventional calculations do not necessarily apply at this fundamental scale or even at much lower energies and larger distances. Then, the cosmological constant problem can be avoided [8, 10].

The indications from Planck and WMAP suggesting the existence of a privileged space direction as predicted by the spinorial space-time, together with the automatic implementation of the LLH law and of the $H\,t = 1$ relation in this approach irrespectively of the apparent sign of the space curvature, are encouraging properties from a phenomenological point of view. Further confirmation of the observational evidence for a privileged space direction is required, and in any case it remains to be checked if the new curvature term can successfully replace the cosmological constant in a global phenomenological description of the Universe.

Other relevant references are [10, 11] (on pre-Big Bang) and [12] (on ultra-high energy cosmic rays).

References

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